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Abstract

We show that the far fields generated by a source inside or near a microparticle can be obtained readily by the use of the reciprocity theorem along with the internal or near fields generated by plane-wave illumination. The method is useful for solving problems for which the scattered fields generated with plane-wave illumination have already been obtained. We illustrate the method for the case of a homogeneous sphere, and then apply it to the problem of emission from a dipole inside a sphere near a plane interface.

Contents

1. Introduction

Methods for obtaining the radiation from a source inside or near a microparticle are needed for a variety of applications: e.g., in modeling the fluorescence [1–4], Raman [5,6], lasing [7–9], or nonlinear emission [10,11] from molecules inside or near scattering objects such as homogeneous [12,13] or layered [14] spheres, spheroids [15], cylinders [16], microdisks [8], radially inhomogeneous bodies [17], particles with complex structures [18], or particles near surfaces [19–33]. Methods for modeling emission from polarization sources inside particles are also needed for some techniques [34] for calculating light scattering by inhomogeneities inside particles [35,36].

The problem of emission from a molecule or other point source inside or near a microparticle can be modeled if we treat each point source as a time harmonic ($\exp(-i\omega t)$ time variation) dipole $\mathbf{p}(\mathbf{r}_b)$ at $\mathbf{r}_b$, which generates an electric field $\mathbf{E}(\mathbf{r}_a)$ at $\mathbf{r}_a$. We can model emission from molecules with nonzero emission linewidths by integrating the electric fields over emission frequencies [3]. The generated fields are related to the dipole source as follows:

$$\begin{bmatrix} E_1(\mathbf{r}_a) \\ E_2(\mathbf{r}_a) \\ E_3(\mathbf{r}_a) \end{bmatrix} = \omega^2\mu \begin{bmatrix} G_{11}(\mathbf{r}_a,\mathbf{r}_b) & G_{12}(\mathbf{r}_a,\mathbf{r}_b) & G_{13}(\mathbf{r}_a,\mathbf{r}_b) \\ G_{21}(\mathbf{r}_a,\mathbf{r}_b) & G_{22}(\mathbf{r}_a,\mathbf{r}_b) & G_{23}(\mathbf{r}_a,\mathbf{r}_b) \\ G_{31}(\mathbf{r}_a,\mathbf{r}_b) & G_{32}(\mathbf{r}_a,\mathbf{r}_b) & G_{33}(\mathbf{r}_a,\mathbf{r}_b) \end{bmatrix} \begin{bmatrix} p_1(\mathbf{r}_b) \\ p_2(\mathbf{r}_b) \\ p_3(\mathbf{r}_b) \end{bmatrix}, \quad (1)$$

where the matrix (typically labeled $\bar{\mathbf{G}}(\mathbf{r}_a,\mathbf{r}_b)$) is the dyadic Green function [14,37],* and where the ω-dependence of $\mathbf{E}$, $\mathbf{G}$, and $\mathbf{p}$ is suppressed. We have

*Another common way to write the Green function relation is

$$\mathbf{E}(\mathbf{r}) = \omega^2\mu \int_{V'} \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')\mathbf{P}(\mathbf{r}')\, dv',$$

where $\mathbf{P}(\mathbf{r}') = -i\omega\mathbf{J}(\mathbf{r}')$. The dipole moment $\mathbf{p}(\mathbf{r}')$ of the source is related to the polarization per unit volume, $\mathbf{P}(\mathbf{r}')$ by

$$\mathbf{p}(\mathbf{r}') = \int_{V'} \mathbf{P}(\mathbf{r}')\, dv'.$$

We use the notation of individual dipoles because we have been modeling radiation from individual molecules.

assumed that the permeability μ is uniform.* Although equation (1) is valid for general $\mathbf{r}_a$ and $\mathbf{r}_b$, in this report we treat only the case in which $\mathbf{r}_a$ is far from the particle, and $\mathbf{r}_b$ is inside, on, or near it.

The Green function of equation (1) obeys the reciprocity relation [14,38,39],†

$$\bar{\mathbf{G}}(\mathbf{r}_a, \mathbf{r}_b) = \bar{\mathbf{G}}^T(\mathbf{r}_b, \mathbf{r}_a), \tag{2}$$

where T indicates transpose. Given a solution to a scattering problem with a source at point $\mathbf{r}_a$, we can use the reciprocity of the Green function to obtain [13] or verify [14] Green-function solutions to scattering/emission problems for sources in other regions. Beginning with the Green function for a source near a sphere, we can use reciprocity to obtain the solutions for the fields generated by an incident plane wave [37].

In this report we describe a simple, reciprocity-based method for obtaining the far-field Green function (the Green function for fields far from an object) for a source inside or near a microparticle or other scattering object, when the solutions for the fields generated by an incident plane wave are known. This far-field Green function differs from the complete Green function for an emission problem in that several elements of the complete Green function are not specified. The far-field Green function is often all that is required because it is typically all that can be detected, for example, by a lens-detector system located far from the particle. A main benefit of the approach is that the far-field Green function can be obtained in a simple manner from existing solutions for the fields generated with plane-wave excitation. Such fields have been obtained and implemented in computer codes for a variety of scattering objects (e.g., homogeneous and layered spheres and spheroids, spheres with continuously variable refractive index, finite cylinders, objects with axisymmetric surfaces described by Chebyshev polynomials, cylinders, and particles on or near plane interfaces [21–26]). Another benefit of the approach is that we can transfer some of the understanding/intuition developed for internal and near fields of spheres [40–42] and cylinders [42] to the emission problem. With the reciprocity relations for plane waves and far fields, the intuition developed with ray-optics or other methods can more readily be used to assist in understanding the problem of emission from a source inside a particle [3]. We can also use known solutions for plane-wave incidence to validate the far-field limits of newly developed Green functions and their computer implementations.

*The assumption of a uniform permeability is valid for the problems we want to model, which are at optical frequencies.

†See Chew [14], pp 410–411. The relation for regions with varying μ is $\bar{\mathbf{G}}(\mathbf{r}_a, \mathbf{r}_b)\mu(\mathbf{r}_b) = \bar{\mathbf{G}}^T(\mathbf{r}_b, \mathbf{r}_a)\mu(\mathbf{r}_a)$.

In section 2, we show how the Green function for the fields far from a microparticle can be obtained from expressions for the fields generated inside or near a microparticle by an incident plane wave. In section 3, we illustrate how this method works for the case of a sphere, a well-studied particle for which the Green function is known. In section 4, we apply the method to obtaining the Green function for a dipole inside a sphere on or near a conducting surface, a problem for which (so far as we know) solutions have not yet been derived. This example illustrates how readily the desired expressions are obtained from the solution to the plane-wave-incidence problem. Section 5 summarizes the paper.

2. Green Function from Reciprocity

We assume that the solution to the problem of a plane wave illuminating the particle is available. Our goal is to obtain that part of $\bar{\mathbf{G}}(\mathbf{r}_a, \mathbf{r}_b)$ required for writing the fields far from the dipole and particle.

We begin by writing the plane-wave-illumination problem in matrix form:

$$\begin{bmatrix} E_1(\mathbf{r}_b) \\ E_2(\mathbf{r}_b) \\ E_3(\mathbf{r}_b) \end{bmatrix} = \begin{bmatrix} F_{11}(\mathbf{r}_b, \mathbf{r}_a) & F_{12}(\mathbf{r}_b, \mathbf{r}_a) & F_{13}(\mathbf{r}_b, \mathbf{r}_a) \\ F_{21}(\mathbf{r}_b, \mathbf{r}_a) & F_{22}(\mathbf{r}_b, \mathbf{r}_a) & F_{23}(\mathbf{r}_b, \mathbf{r}_a) \\ F_{31}(\mathbf{r}_b, \mathbf{r}_a) & F_{32}(\mathbf{r}_b, \mathbf{r}_a) & F_{33}(\mathbf{r}_b, \mathbf{r}_a) \end{bmatrix} \begin{bmatrix} E_o\mathbf{e}_o \cdot \mathbf{i}_1(\mathbf{r}_a) \\ E_o\mathbf{e}_o \cdot \mathbf{i}_2(\mathbf{r}_a) \\ E_o\mathbf{e}_o \cdot \mathbf{i}_3(\mathbf{r}_a) \end{bmatrix}, \quad (3)$$

where $E_1(\mathbf{r}_b)$, $E_2(\mathbf{r}_b)$, and $E_3(\mathbf{r}_b)$ are the field components at position $\mathbf{r}_b$ inside or near the particle, and the incident plane-wave field is given by

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = E_o\mathbf{e}_o e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (4)$$

where $\mathbf{k}$ is the propagation vector and the unit vector $\mathbf{e}_o$ is perpendicular to $\mathbf{k}$. We write the unit vectors as $\mathbf{i}_j(\mathbf{r}_a)$ to emphasize that each is evaluated at $\mathbf{r}_a$. An example of F_{ij} for axisymmetric particles analyzed in a spherical coordinate system is given in appendix A.

The incident plane wave can be generated to any degree of accuracy by a time-harmonic dipole polarization source, $\mathbf{p}(\mathbf{r}_a) = \mathbf{e}_o p(\mathbf{r}_a)$, with $\mathbf{e}_o$ perpendicular to $\mathbf{r}_a$, when the dipole is sufficiently far from the object (i.e., $|\mathbf{r}_a| \gg \lambda$, and $|\mathbf{r}_a|$ is many times larger than the object). (The surface on which the outgoing boundary conditions are applied for the Green-function solutions is many times further from the object than $|\mathbf{r}_a|$.) Near the particle, the field of the dipole is [43]

$$\mathbf{E}(\mathbf{r}) = \frac{\omega^2\mu}{4\pi} \frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_a)}}{|\mathbf{r}-\mathbf{r}_a|} p(\mathbf{r}_a)\mathbf{e}_o. \quad (5)$$

Comparing equations (4) and (5), we see that the amplitude of the plane wave generated by the source $\mathbf{p}(\mathbf{r}_a)$ is approximately

$$E_o = \frac{\omega^2\mu e^{-i\mathbf{k}\cdot\mathbf{r}_a}}{4\pi r_a} p(\mathbf{r}_a), \quad (6)$$

where $|\mathbf{r}-\mathbf{r}_a|$ in the denominator is replaced by $r_a = |\mathbf{r}_a|$ because the points $\mathbf{r}$ are near the particle and because $|\mathbf{r}| \ll |\mathbf{r}_a|$.

Using equation (6) for E_o, we can rewrite equation (3) as

$$\begin{bmatrix} E_1(\mathbf{r}_b) \\ E_2(\mathbf{r}_b) \\ E_3(\mathbf{r}_b) \end{bmatrix} = \omega^2\mu \begin{bmatrix} G_{11}(\mathbf{r}_b,\mathbf{r}_a) & G_{12}(\mathbf{r}_b,\mathbf{r}_a) & G_{13}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{21}(\mathbf{r}_b,\mathbf{r}_a) & G_{22}(\mathbf{r}_b,\mathbf{r}_a) & G_{23}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{31}(\mathbf{r}_b,\mathbf{r}_a) & G_{32}(\mathbf{r}_b,\mathbf{r}_a) & G_{33}(\mathbf{r}_b,\mathbf{r}_a) \end{bmatrix} \begin{bmatrix} p_1(\mathbf{r}_a) \\ p_2(\mathbf{r}_a) \\ p_3(\mathbf{r}_a) \end{bmatrix}, \tag{7}$$

where

$$G_{ij}(\mathbf{r}_b,\mathbf{r}_a) = \frac{e^{-i\mathbf{k}\cdot\mathbf{r}_a}}{4\pi r_a} F_{ij}(\mathbf{r}_b,\mathbf{r}_a). \tag{8}$$

Equations (7) and (8) provide the Green function for the fields inside an object excited by a dipole far from the object. Then, using the reciprocity relation, equation (2), we obtain the desired elements of $\bar{\mathbf{G}}(\mathbf{r}_a,\mathbf{r}_b)$ generated by a source at $\mathbf{r}_b$ inside or near the particle:

$$\begin{bmatrix} E_1(\mathbf{r}_a) \\ E_2(\mathbf{r}_a) \\ E_3(\mathbf{r}_a) \end{bmatrix} = \omega^2\mu \begin{bmatrix} G_{11}(\mathbf{r}_b,\mathbf{r}_a) & G_{21}(\mathbf{r}_b,\mathbf{r}_a) & G_{31}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{12}(\mathbf{r}_b,\mathbf{r}_a) & G_{22}(\mathbf{r}_b,\mathbf{r}_a) & G_{32}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{13}(\mathbf{r}_b,\mathbf{r}_a) & G_{23}(\mathbf{r}_b,\mathbf{r}_a) & G_{33}(\mathbf{r}_b,\mathbf{r}_a) \end{bmatrix} \begin{bmatrix} p_1(\mathbf{r}_b) \\ p_2(\mathbf{r}_b) \\ p_3(\mathbf{r}_b) \end{bmatrix}. \tag{9}$$

It must be emphasized that the above Green function is valid only for field points $\mathbf{r}_a$ far from the particle. For $\mathbf{r}_a$ close to the particle, some or all of the G_{ij} would have additional terms that would decay with distance from the particle.

If a spherical coordinate system is chosen, then $\mathbf{r}_a$ is far from the particle, on a line going from the origin in the $-\mathbf{k}$ (or $\mathbf{i}_r$ direction), and $p_1(\mathbf{r}_a)$ for the incident plane wave is zero; thus, the plane-wave-illumination problem can be written as

$$\begin{bmatrix} E_r(\mathbf{r}_b) \\ E_\theta(\mathbf{r}_b) \\ E_\phi(\mathbf{r}_b) \end{bmatrix} = \begin{bmatrix} U^F_{11}(\mathbf{r}_b,\mathbf{r}_a) & F_{12}(\mathbf{r}_b,\mathbf{r}_a) & F_{13}(\mathbf{r}_b,\mathbf{r}_a) \\ U^F_{21}(\mathbf{r}_b,\mathbf{r}_a) & F_{22}(\mathbf{r}_b,\mathbf{r}_a) & F_{23}(\mathbf{r}_b,\mathbf{r}_a) \\ U^F_{31}(\mathbf{r}_b,\mathbf{r}_a) & F_{32}(\mathbf{r}_b,\mathbf{r}_a) & F_{33}(\mathbf{r}_b,\mathbf{r}_a) \end{bmatrix} \begin{bmatrix} 0 \\ E_o\mathbf{e}_o\cdot\mathbf{i}_\theta \\ E_o\mathbf{e}_o\cdot\mathbf{i}_\phi \end{bmatrix}, \tag{10}$$

where we write the $F_{i1}(\mathbf{r}_b,\mathbf{r}_a)$ as $U^F_{ij}(\mathbf{r}_b,\mathbf{r}_a)$ to emphasize that they are unspecified and unneeded for the internal/near fields at $\mathbf{r}_b$. The $F_{i2}(\mathbf{r}_b,\mathbf{r}_a)$ and $F_{i3}(\mathbf{r}_b,\mathbf{r}_a)$ are known. The desired Green function, obtained with equations (8) and (2), is

$$\begin{bmatrix} 0 \\ E_\theta(\mathbf{r}_a) \\ E_\phi(\mathbf{r}_a) \end{bmatrix} = \omega^2\mu \begin{bmatrix} U^G_{11}(\mathbf{r}_b,\mathbf{r}_a) & U^G_{21}(\mathbf{r}_b,\mathbf{r}_a) & U^G_{31}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{12}(\mathbf{r}_b,\mathbf{r}_a) & G_{22}(\mathbf{r}_b,\mathbf{r}_a) & G_{32}(\mathbf{r}_b,\mathbf{r}_a) \\ G_{13}(\mathbf{r}_b,\mathbf{r}_a) & G_{23}(\mathbf{r}_b,\mathbf{r}_a) & G_{33}(\mathbf{r}_b,\mathbf{r}_a) \end{bmatrix} \begin{bmatrix} p_r(\mathbf{r}_b) \\ p_\theta(\mathbf{r}_b) \\ p_\phi(\mathbf{r}_b) \end{bmatrix}, \tag{11}$$

where $G_{11}(\mathbf{r}_b, \mathbf{r}_a)$, $G_{21}(\mathbf{r}_b, \mathbf{r}_a)$, and $G_{31}(\mathbf{r}_b, \mathbf{r}_a)$ are written as $U_{j1}^G(\mathbf{r}_b, \mathbf{r}_a)$, similar to the $U_{i1}^F(\mathbf{r}_b, \mathbf{r}_a)$ of equation (10), because the values are unspecified and not needed for the far-field solutions at $\mathbf{r}_a$. The above expression is valid for complex ω, and so may be useful for treating problems in terms of quasinormal modes [44].

The relations given in equations (9) and (11) are key results of this report. In particular, in the spherical coordinate system commonly used for particle scattering problems, equation (11) indicates how the known internal field solutions (the F_{nm}) specify the far fields from an arbitrary dipole source inside the microparticle. The understanding of the spatial variations of the F_{nm} (proportional to the G_{mn}) for some microparticles (spheres [40–42] and cylinders [42]) can now more readily be used to visualize and understand the emission problem. Distributions of fluorescence collected from oriented dipoles inside a sphere have been previously shown [3]. Although the previous study noted that the emission problem was related to the incident field problem by reciprocity, the specific relations shown in equation (11) were not known at that time.

3. Example: Fields from a Source in a Homogeneous Sphere

To illustrate the above method for a well-studied particle (a homogeneous sphere), we compare the fields at $\mathbf{r}_a$ on the $-z$ axis far from the particle generated by a dipole inside the sphere and lying in the $\phi = 0$ (i.e., $x - z$) plane. In this case, we use the spherical coordinate system; i.e., $\mathbf{i}_1 = \mathbf{i}_{\hat{r}}$ is the unit vector in the radial direction, $\mathbf{i}_2 = \mathbf{i}_\theta$, and $\mathbf{i}_3 = \mathbf{i}_\phi$. Expressions for the fields emitted by a source inside a sphere are well known [12]. The fluorescence collected from a dipole inside a sphere, calculated as a function of dipole position, has been illustrated elsewhere [3].

3.1 Using Reciprocity

For points in the $\phi = 0$ plane, $F_{13}(\mathbf{r}_b, \mathbf{r}_a)$, $F_{23}(\mathbf{r}_b, \mathbf{r}_a)$, and $F_{32}(\mathbf{r}_b, \mathbf{r}_a)$ are zero (see app B), and equation (3) reduces to

$$\begin{bmatrix} E_r(\mathbf{r}_b) \\ E_\theta(\mathbf{r}_b) \\ E_\phi(\mathbf{r}_b) \end{bmatrix} = \begin{bmatrix} U_{11}^F(\mathbf{r}_b, \mathbf{r}_a) & F_{12}(\mathbf{r}_b, \mathbf{r}_a) & 0 \\ U_{21}^F(\mathbf{r}_b, \mathbf{r}_a) & F_{22}(\mathbf{r}_b, \mathbf{r}_a) & 0 \\ U_{31}^F(\mathbf{r}_b, \mathbf{r}_a) & 0 & F_{33}(\mathbf{r}_b, \mathbf{r}_a) \end{bmatrix} \begin{bmatrix} 0 \\ E_o \mathbf{e}_o \cdot \mathbf{i}_\theta(\mathbf{r}_a) \\ E_o \mathbf{e}_o \cdot \mathbf{i}_\phi(\mathbf{r}_a) \end{bmatrix} . \tag{12}$$

The desired Green function, as in equations (9) and (11), is of the form

$$\begin{bmatrix} E_r(\mathbf{r}_a) \\ E_\theta(\mathbf{r}_a) \\ E_\phi(\mathbf{r}_a) \end{bmatrix} = \omega^2 \mu \begin{bmatrix} U_{11}^G(\mathbf{r}_b, \mathbf{r}_a) & U_{21}^G(\mathbf{r}_b, \mathbf{r}_a) & U_{31}^G(\mathbf{r}_b, \mathbf{r}_a) \\ G_{12}(\mathbf{r}_b, \mathbf{r}_a) & G_{22}(\mathbf{r}_b, \mathbf{r}_a) & 0 \\ 0 & 0 & G_{33}(\mathbf{r}_b, \mathbf{r}_a) \end{bmatrix} \begin{bmatrix} p_r(\mathbf{r}_b) \\ p_\theta(\mathbf{r}_b) \\ p_\phi(\mathbf{r}_b) \end{bmatrix} . \tag{13}$$

We consider the case of a ϕ-polarized dipole in which $p_1 = p_2 = 0$, and $p_3(\mathbf{r}_b) = p_\phi$. Using the Green function from reciprocity (eq (13)) and equation (8), we obtain $E_\theta(\mathbf{r}_a) = 0$, and

$$E_\phi(\mathbf{r}_a) = \omega^2 \mu G_{33}(\mathbf{r}_b, \mathbf{r}_a) p_\phi = \omega^2 \mu p_\phi \frac{e^{ikr_a}}{4\pi r_a} F_{33}(\mathbf{r}_b, \mathbf{r}_a), \tag{14}$$

where $\mathbf{k} \cdot \mathbf{r}_a = -kr_a$ because $\mathbf{r}_a$ is on the $-z$ axis.

Using the F_{33} given by equation (A-6) with $m = 1$, we obtain from equation (14)

$$
\begin{aligned}
E_\phi(\mathbf{r}_a) &= \omega^2 \mu p_\phi \frac{e^{ikr_a}}{4\pi r_a} \\
&\times \sum_n \left[-j_n(\eta k r_b) \frac{d}{d\theta_b} P_n^1(\cos\theta_b) c_{eln} + \frac{1}{\eta k r_b} \frac{d}{dr_b} \left[r_b j_n(\eta k r_b) \right] \frac{P_n^1(\cos\theta_b)}{\sin\theta_b} d_{oln} \right],
\end{aligned} \tag{15}
$$

where from equations (4.3) and (4.5) of Barber and Hill [15]

$$
c_{eln} = -i^n \frac{2n+1}{n(n+1)} \frac{i}{x j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)}, \tag{16}
$$

$$
d_{oln} = -i^{n+1} \frac{2n+1}{n(n+1)} \frac{\eta i}{\eta^2 x j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)}, \tag{17}
$$

where x is the size parameter of the sphere and η is its refractive index. With equations (16) and (17), equation (15) can be written

$$
\begin{aligned}
E_\phi(\mathbf{r}_a) &= \frac{e^{ikr_a}}{kr_a} \sum_n \frac{\omega^2 \mu k p_\phi}{4\pi} \frac{(2n+1)}{n(n+1)} \left[\frac{i^{n+1} j_n(\eta k r_b) \frac{d}{d\theta_b} P_n^1(\cos\theta_b)}{x j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)} \right. \\
&\quad \left. + \frac{\frac{i^n}{kr_b} \frac{d}{dr_b} \left[r_b j_n(\eta k r_b) \right] \frac{P_n^1(\cos\theta_b)}{\sin\theta_b}}{\eta^2 x j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)} \right].
\end{aligned} \tag{18}
$$

3.2 Using the Complete Green-Function Solution

We verify our results by comparing them with results obtained in the traditional boundary-value fashion: i.e., starting out with a radiating dipole within the sphere, satisfying the boundary conditions at the sphere surface and finding the resulting scattered far field in the $\theta_a = \pi$ direction. Using the complete Green-function solution for a source inside a sphere (see app B) with $p_3(\mathbf{r}_b) = p_\phi$ and $p_1 = p_2 = 0$, we obtain the scattered field coefficients as

$$
f\nu^G = -\omega^2 \mu p_\phi \frac{k}{\pi x} \frac{\mathbf{i}_\phi \cdot \mathbf{M}\nu^1(\eta k \mathbf{r}_b)}{j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' h_n^{(1)}(x)}, \tag{19}
$$

by using equations (B-4) and (B-7), and

$$g\nu^G = -\omega^2 \mu p_\phi \frac{k}{\pi x} \frac{\eta \mathbf{i}_\phi \cdot \mathbf{N}\nu^1(\eta k \mathbf{r}_b)}{\eta^2 j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' h_n^{(1)}(x)}, \tag{20}$$

by using equations (B-5) and (B-8).

From the definitions of $\mathbf{M}\nu^1(\eta k \mathbf{r}_b)$ and $\mathbf{N}\nu^1(\eta k \mathbf{r}_b)$, the $f\nu^G$ and $g\nu^G$ simplify to $f^G_{omn} = g^G_{emn} = 0$, and

$$f^G_{emn} = \frac{\omega^2 \mu k p_\phi}{\pi x} \frac{j_n(\eta k r_b) \dfrac{d}{d\theta_b} P_n^m(\cos\theta_b)}{j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' h_n^{(1)}(x)}, \tag{21}$$

$$g^G_{omn} = \frac{-\omega^2 \mu p_\phi}{\pi x r_b} \frac{\dfrac{d}{dr_b}[r j_n(\eta k r_b)] \dfrac{P_n^m(\cos\theta_b)}{\sin\theta_b}}{\eta^2 j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' h_n^{(1)}(x)}. \tag{22}$$

Now, by restricting the field points $\mathbf{r}_a$ to the z axis where the contributions for $m \neq 1$ are zero, and restricting the source points $\mathbf{r}_b$ to the $\phi = 0$ plane, noting that

$$\left.\frac{P_n^1(\cos\theta_a)}{\sin\theta_a}\right|_{\theta_a=\pi} = (-1)^{n+1} \frac{n(n+1)}{2}, \tag{23}$$

$$\left.\frac{d}{d\theta_a} P_n^1(\cos\theta_a)\right|_{\theta_a=\pi} = (-1)^n \frac{n(n+1)}{2} \tag{24}$$

(because $\theta_a = \pi$ at the field point), and using the far-field expansion of the spherical Hankel function,

$$h_n^{(1)}(kr) = \frac{i^{-(n+1)}}{kr} e^{ikr}, \tag{25}$$

we obtain

$$\mathbf{M}_{e1n}(k\mathbf{r})|_{\theta=\pi} = i^{n+1} \frac{n(n+1)}{2} \frac{e^{ikr}}{kr} \mathbf{i}_\phi, \tag{26}$$

$$\mathbf{N}_{o1n}(k\mathbf{r})|_{\theta=\pi} = -i^n \frac{n(n+1)}{2} \frac{e^{ikr}}{kr} \mathbf{i}_\phi. \tag{27}$$

Then using equation (B-6), we obtain $E^s_\theta = 0$, and

$$E^s_\phi = \frac{e^{ikr_a}}{kr_a} \sum_n \frac{n(n+1)}{2} D_{1n} \left[i^{(n+1)} f_{e1n} + i^n g_{o1n} \right], \tag{28}$$

which is equivalent to

$$
\begin{aligned}
E_\phi^s &= \frac{e^{ikr_a}}{kr_a}\sum_n \frac{\omega^2\mu k p_\phi}{4\pi}\frac{(2n+1)}{n(n+1)}\left[\frac{i^{(n+1)}j_n(\eta k r_b)\frac{d}{d\theta_b}P_n^1(\cos\theta_b)}{xj_n(\eta x)[xh_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)}\right. \\
&+ \left.\frac{\frac{i^n}{kr_b}\frac{d}{dr_b}[r_b j_n(\eta k r_b)]\frac{P_n^1(\cos\theta_b)}{\sin\theta_b}}{\eta^2 x j_n(\eta x)[xh_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)}\right].
\end{aligned}
\tag{29}
$$

The expressions for the electric field that we obtained using reciprocity (eq (18)) and using the complete solution (eq (29)) are the same.

4. Example: Fields from Source in Homogeneous Sphere near a Plane Conducting Interface

Fluorescence and Raman emission have been used in characterizing particles on, inside, or near a surface, e.g., a biological cell or spore on a filter, or a contaminant particle on a silicon wafer. Solutions have been described for the fields scattered by a sphere in close proximity to a plane interface and illuminated with a plane wave [21–26]. However, as far as we know, solutions for the fields emitted from a dipole inside a sphere on a plane surface have not been described. Here we use the reciprocity theorem and a known solution for the fields generated in a sphere with plane-wave excitation to write the solution for the far fields generated by a dipole emitting inside a sphere on or near a plane surface. To obtain the F_{ij} required for equation (3), we use the derivation of Videen [26] who presents a solution to the fields of a particle on or near a perfectly conducting plane surface. The vector spherical harmonics used in that article [26] and in this section are normalized:

$$\begin{aligned}
\tilde{\mathbf{M}}_{nm}^{(\rho)} &= \hat{\theta}\frac{im}{\sin\theta}z_n^{(\rho)}(kr)\tilde{P}_n^m(\cos\theta)e^{im\phi} - \hat{\phi}z_n^{(\rho)}(kr)\frac{d}{d\theta}\tilde{P}_n^m(\cos\theta)e^{im\phi}, \qquad (30)\\
\tilde{\mathbf{N}}_{nm}^{(\rho)} &= \hat{r}\frac{1}{kr}z_n^{(\rho)}(kr)n(n+1)\tilde{P}_n^m(\cos\theta)e^{im\phi} + \hat{\theta}\frac{1}{kr}\frac{d}{dr}\left[rz_n^{(\rho)}(kr)\right]\frac{d}{d\theta}\tilde{P}_n^m(\cos\theta)e^{im\phi}\\
&+ \hat{\phi}\frac{1}{kr}\frac{d}{dr}\left[rz_n^{(\rho)}(kr)\right]\frac{im}{\sin\theta}\tilde{P}_n^m(\cos\theta)e^{im\phi}.
\end{aligned}$$

Although it is usually not appropriate to use multiple definitions of the vector spherical harmonics in one paper, here the multiple definitions help illustrate the generality of the approach stated in section 2.

We solved the plane-wave-incidence problem by expanding the incident plane wave, the scattered fields, and fields interior to the sphere in vector spherical harmonics. In addition to these fields, there is an interaction field that scatters from the sphere, reflects from the plane surface, and illuminates the sphere again. We determine the field coefficients by forcing the boundary conditions at the interfaces of the sphere and plane surface to be satisfied simultaneously.

The fields inside the homogeneous sphere are expanded as [26]

$$E_1^{\rm int}(\eta kr) = \sum_{n,m} e_{nm}^{(1)} \tilde{M}_{nm}^{(1)}(\eta kr) + e_{nm}^{(2)} \tilde{N}_{nm}^{(1)}(\eta kr), \tag{31}$$

where $e_{nm}^{(j)}$ are the interior field coefficients. These coefficients are expressed in terms of the known scattering, interaction, and incident field coefficients, $b_{nm}^{(j)}$, $c_{nm}^{(j)}$, and $a_{nm}^{(j)}$, as

$$e_{nm}^{(1)} j_n(\eta kr) = a_{nm}^{(1)} j_n(ka) + b_{nm}^{(1)} h_n(ka) + c_{nm}^{(1)} j_n(ka), \tag{32}$$

$$\mathrm{m} e_{nm}^{(2)} j_n(\eta kr) = a_{nm}^{(2)} j_n(ka) + b_{nm}^{(2)} h_n(ka) + c_{nm}^{(2)} j_n(ka). \tag{33}$$

We can express equations (31) to (33) in the Green-function formalism of section 2 by writing out the vector spherical harmonics in equation (31), and writing the fields in terms of the F_{ij} of equation (3) as

$$F_{12}(\mathbf{r}_b, \mathbf{r}_a) = \sum_{n,m} e_{nm}^{(2)TM} \frac{1}{\eta k r_b} z_n^{(1)}(\eta k r_b) n(n+1) \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \tag{34}$$

$$F_{13}(\mathbf{r}_b, \mathbf{r}_a) = \sum_{n,m} e_{nm}^{(2)TE} \frac{1}{\eta k r_b} z_n^{(1)}(\eta k r_b) n(n+1) \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \tag{35}$$

$$\begin{aligned} F_{22}(\mathbf{r}_b, \mathbf{r}_a) = {} & \sum_{n,m} e_{nm}^{(1)TM} \frac{im}{\sin\theta_b} z_n^{(1)}(\eta k r_b) \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b} + \\ & e_{nm}^{(2)TM} \frac{1}{\eta k r_b} \frac{d}{dr_b} \left[r_b z_n^{(1)}(\eta k r_b) \right] \frac{d}{d\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \end{aligned} \tag{36}$$

$$\begin{aligned} F_{23}(\mathbf{r}_b, \mathbf{r}_a) = {} & \sum_{n,m} e_{nm}^{(1)TE} \frac{im}{\sin\theta_b} z_n^{(1)}(\eta k r_b) \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b} + \\ & e_{nm}^{(2)TE} \frac{1}{\eta k r_b} \frac{d}{dr_b} \left[r_b z_n^{(1)}(\eta k r_b) \right] \frac{d}{d\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \end{aligned} \tag{37}$$

$$\begin{aligned} F_{32}(\mathbf{r}_b, \mathbf{r}_a) = {} & \sum_{n,m} -e_{nm}^{(1)TM} z_n^{(1)}(\eta k r_b) \frac{d}{d\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b} + \\ & e_{nm}^{(2)TM} \frac{1}{\eta k r_b} \frac{d}{dr_b} \left[r_b z_n^{(1)}(\eta k r_b) \right] \frac{im}{\sin\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \end{aligned} \tag{38}$$

$$\begin{aligned} F_{33}(\mathbf{r}_b, \mathbf{r}_a) = {} & \sum_{n,m} -e_{nm}^{(1)TE} z_n^{(1)}(\eta k r_b) \frac{d}{d\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b} + \\ & e_{nm}^{(2)TE} \frac{1}{\eta k r_b} \frac{d}{dr_b} \left[r_b z_n^{(1)}(\eta k r_b) \right] \frac{im}{\sin\theta_b} \tilde{P}_n^m(\cos\theta_b) e^{im\phi_b}, \end{aligned} \tag{39}$$

where the superscripts TE and TM, on the internal field coefficients, refer to the polarization state of the incident plane wave. The desired Green function is then given directly by equation (11) with $G_{ij}(\mathbf{r}_b, \mathbf{r}_a)$ obtained from $F_{ij}(\mathbf{r}_b, \mathbf{r}_a)$ as described in equation (8). Thus, the scattered field components

resulting from a dipole within a sphere near a perfectly conducting surface are obtained with equation (31).

Although the above was derived for the particular problem of a sphere above a surface, the preceding expressions for the $F_{ij}(\mathbf{r}_b, \mathbf{r}_a)$ and the $G_{ij}(\mathbf{r}_b, \mathbf{r}_a)$ are valid for any particle for which the internal field coefficients of equation (31), $e_{nm}^{(1)}$ and $e_{nm}^{(2)}$, are known. For a dipole outside but near the particle, equations (34) to (39) may also be used, where the internal field coefficients $e_{nm}^{(j)T*}$ are now replaced by the scattered field coefficients, and the Bessel functions are replaced by the appropriate Hankel functions. The particular case of a sphere near a substrate is slightly more complicated when the dipole is outside the sphere, because the interaction field must also be included with the scattered field. Finally, although we have used normalized vector spherical harmonics, these equations may also be applied to coefficients derived with unnormalized vector spherical harmonics by a simple replacement of the normalized associated Legendre polynomials, $\tilde{P}_n^m(\cos\theta_b)$, with the associated Legendre polynomials, $P_n^m(\cos\theta_b)$.

5. Summary

This report is based on the following three observations: (1) The Green function relating a source and a scattered field obeys a reciprocity relation [14,38]. (2) Solutions for the fields inside or near a variety of scattering objects have been obtained for plane-wave excitation but not more general sources. (3) For many problems in which a molecule or polarization source emits inside or near a scattering object, it is only the far fields that are measured or are of interest; for these problems, a far-field Green function that only specifies the fields far from the particle is sufficient.

The key point of this report is that reciprocity and known solutions for the fields generated in a particle by plane-wave incident fields can be readily used to find the far fields emitted by a source inside or near the particle. Although only a partial Green function is known from the plane-wave-incidence problem, that partial Green function and reciprocity are sufficient for specifying the far fields. These key results are given in equations (9) and (11).

In section 4, we illustrate the technique by applying it to a particle for which the solution is well known: a homogeneous sphere in a homogeneous medium. We then demonstrate the power of this technique by applying it to a more complicated problem for which the fields emitted by a dipole in the particle are not known: a dipole located within a sphere near a plane interface. Although the solution was derived for this particular system, the equations given in section 4 can be applied to any system for which the internal or scattering coefficients of an expansion in vector spherical harmonics have been derived.

Another benefit of understanding these reciprocity relations of the internal fields generated by incident plane waves is that the understanding of the internal intensity patterns of particles (obtained from ray-optic analyses, comparisons with Fabry-Perot cavities, etc) can be used to help develop an understanding of emission patterns from sources inside the particle [3].

Appendix A. Example: F_{ij} for Axisymmetric Particles

If the scattering solution is found for an axisymmetric particle with a spherical coordinate system and spherical wave functions, and $\mathbf{r}_b$ is in the $\phi = 0$ plane, then F_{12}, F_{22}, F_{32} are given by the summation over n' of E_r^{int}, E_θ^{int}, E_ϕ^{int} of equation (3.10) of Barber and Hill [15], i.e.,

$$F_{12} = \sum_{n,m} n(n+1)\frac{j_n(\eta k r_b)}{\eta k r_b}\cos m\phi_b \frac{P_n^m(\cos\theta_b)}{\sin\theta_b}\sin\theta_b d_{emn}, \tag{A-1}$$

$$\begin{aligned} F_{22} &= \sum_{n,m} j_n(\eta k r_b)\cos m\phi_b \frac{mP_n^m(\cos\theta_b)}{\sin\theta_b} c_{omn} \\ &+ \frac{1}{\eta k r_b}\frac{d}{dr_b}\left[r_b j_n(\eta k r_b)\right]\cos m\phi \frac{d}{d\theta}P_n^m(\cos\theta_b)\ d_{emn}, \end{aligned} \tag{A-2}$$

$$\begin{aligned} F_{32} &= \sum_{n,m} -j_n(\eta k r_b)\sin m\phi_b \frac{d}{d\theta}P_n^m(\cos\theta_b) c_{omn} \\ &- \frac{1}{\eta k r_b}\frac{d}{dr_b}\left[r_b j_n(\eta k r_b)\right]\sin m\phi_b \frac{mP_n^m(\cos\theta_b)}{\sin\theta_b} d_{emn}, \end{aligned} \tag{A-3}$$

where the c_{emn}, c_{omn}, d_{emn}, and d_{omn} are the internal field coefficients of the particle. The F_{i1} are not specified in the plane-wave-incidence problem. The F_{13}, F_{23}, F_{33} are given by the summation over n' of E_r^{int}, E_θ^{int}, E_ϕ^{int} of equation (3.11) of Barber and Hill [15]:

$$F_{13} = \sum_{n,m} n(n+1)\frac{j_n(\eta k r_b)}{\eta k r_b}\sin m\phi_b \frac{P_n^m(\cos\theta_b)}{\sin\theta_b}\sin\theta_b d_{omn}, \tag{A-4}$$

$$\begin{aligned} F_{23} &= \sum_{n,m} -j_n(\eta k r_b)\sin m\phi_b \frac{mP_n^m(\cos\theta_b)}{\sin\theta_b} c_{emn} \\ &+ \frac{1}{\eta k r_b}\frac{d}{dr_b}\left[r_b j_n(\eta k r_b)\right]\sin m\phi_b \frac{d}{d\theta}P_n^m(\cos\theta_b) d_{omn}, \end{aligned} \tag{A-5}$$

$$\begin{aligned} F_{33} &= \sum_{n,m} -j_n(\eta k r_b)\cos m\phi_b \frac{d}{d\theta}P_n^m(\cos\theta_b) c_{emn} \\ &+ \frac{1}{\eta k r_b}\frac{d}{dr_b}\left[r_b j_n(\eta k r_b)\right]\cos m\phi_b \frac{mP_n^m(\cos\theta_b)}{\sin\theta_b} d_{omn}. \end{aligned} \tag{A-6}$$

If the particle has spherical symmetry, these equations can be simplified, since only the $m = 1$ terms contribute to the scattered fields. The reduced equations for F_{12}, F_{22}, F_{32} are given by the summation over n of E_r^{int}, E_θ^{int}, E_ϕ^{int} of equation (4.33) [15], and F_{13}, F_{23}, F_{33} are given by the summation over n of E_r^{int}, E_θ^{int}, E_ϕ^{int} of equation (4.34).

Appendix B. Fields from a Dipole Inside a Sphere

The Green function for a source inside a sphere has been described [12–14,37]. The total internal field generated by the source inside the sphere is [3]

$$\mathbf{E}^T(\eta k\mathbf{r}) = \mathbf{E}^H(\eta k\mathbf{r}) + \mathbf{E}^{iG}(\eta k\mathbf{r}). \tag{B-1}$$

Here, $\mathbf{E}^H(\eta k\mathbf{r})$ is the electric field at $\mathbf{r}$ radiated by arbitrary polarization sources at $\mathbf{r}'$ (where $\mathbf{r} > \mathbf{r}'$) in a homogeneous region of refractive index η,

$$\mathbf{E}^H(\eta k\mathbf{r}) = \sum_{\nu=1}^{\infty} D\nu[c\nu^H \mathbf{M}\nu^3(\eta k\mathbf{r}) + d\nu^H \mathbf{N}\nu^3(\eta k\mathbf{r})]. \tag{B-2}$$

The superscripts on the vector spherical harmonics refer to the kind of radial function (first or third) used in the expansion, and ν represents the spherical harmonic triple index σ, m, n, where σ is even or odd, n is the mode number, and m is the azimuthal mode number. The normalization constant is

$$D_{mn} = \frac{\epsilon_m(2n+1)(n-m)!}{4n(n+1)(n+m)!}, \tag{B-3}$$

where ϵ_m is equal to 1 for $m = 0$ and equal to 2 for $m > 0$.

The field expansion coefficients $c\nu^H$ and $d\nu^H$ are determined by the strength, position, and orientation of the source polarization $\mathbf{p}(\mathbf{r}_b)$ as

$$c\nu^H = i\omega^2\mu\frac{k\eta}{\pi}\mathbf{p}(\mathbf{r}_b) \cdot \mathbf{M}\nu^1(\eta k\mathbf{r}_b), \tag{B-4}$$

$$d\nu^H = i\omega^2\mu\frac{k\eta}{\pi}\mathbf{p}(\mathbf{r}_b) \cdot \mathbf{N}\nu^1(\eta k\mathbf{r}_b), \tag{B-5}$$

where the superscript H indicates a homogeneous region.

The induced electric fields outside the sphere are expanded as

$$\mathbf{E}^{sG}(k\mathbf{r}) = \sum_{\nu=1}^{\infty} D\nu[f\nu^G \mathbf{M}\nu^3(k\mathbf{r}) + g\nu^G \mathbf{N}\nu^3(k\mathbf{r})], \tag{B-6}$$

where $f\nu^G$ and $g\nu^G$ are termed the "scattered" field expansion coefficients because of their similarity in equation (B-6) to the scattering coefficients of the usual scattering problem. The G in the superscripts of the $c\nu^{iG}$, $d\nu^{iG}$,

$f\nu^G$, and $g\nu^G$ differentiates these internal and scattered coefficients from the field coefficients used with other incident fields.

The scattered field coefficients are

$$f\nu^G = c\nu^H \frac{i}{\eta x j_n(\eta x)[x h_n^{(1)}(x)]' - \eta[\eta x j_n(\eta x)]' x h_n^{(1)}(x)}, \tag{B-7}$$

$$g\nu^G = d\nu^H \frac{i}{\eta^2 x j_n(\eta x)[x h_n^{(1)}(x)]' - [\eta x j_n(\eta x)]' x h_n^{(1)}(x)}, \tag{B-8}$$

where x is the size parameter and η is the refractive index of the host sphere, $j_n(\eta x)$ is the spherical Bessel function, and $h_n^{(1)}(x)$ is the spherical Hankel function of the first kind. All derivatives (denoted by the primes) are with respect to the argument.

In our earlier work describing the fields from a dipole inside a sphere, the sign of the coefficients of the induced internal transverse magnetic fields (the $d\nu^{iG}$ in eq (A9) of Hill et al. [3] and in eq (12) of Hill et al. [34]) should have been negative. These $d\nu^{iG}$ coefficients are not used here, nor were they used in the previous references [3,34]. However, the sign error is important for anyone verifying the above expressions.

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We show that the far fields generated by a source inside or near a microparticle can be obtained readily by using the reciprocity theorem along with the internal or near fields generated by plane-wave illumination. The method is useful for solving problems for which the scattered fields generated with plane-wave illumination have already been obtained. We illustrate the method for the case of a homogeneous sphere, and then apply it to the problem of emission from a dipole inside a sphere near a plane interface.

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